



## What are the Limits?

Consider a function  $f(x)$  to be defined in an open interval near the number “ $c$ ” (need not be at  $c$ ). If, as  $x$  approaches “ $c$ ” from the left and right side of “ $c$ ”,  $f(x)$  approaches a particular number “ $L$ ”, then “ $L$ ” is called the limit of  $f(x)$  as  $x$  approaches  $c$ .

Mathematically, the limit is represented as:

$\lim_{x \rightarrow c} f(x) = L$  (read as “limit of  $f(x)$ , as  $x$  approaches  $c$  is  $L$ )

## Types of limit

There are multiple [types of limits](#), but some common types of limits are given below.

1. **Limit at a point:** This refers to finding the limit of a function as  $x$  approaches a specific point. For example  $\lim_{x \rightarrow c} f(x)$ .
2. **Infinite limit:** In this type of limit, the limit of a function tends to positive or negative infinity as  $x$  approaches a certain point. For example  $\lim_{x \rightarrow c} f(x) = \pm \infty$ .
3. **Limit at Infinity:** This involves finding the limit of a function as  $x$  approaches positive or negative infinity. For example  $\lim_{x \rightarrow \pm \infty} f(x)$
4. **One-sided limit:** It represents the behavior of the function as the input values approach a specific point from the left ( $\lim_{x \rightarrow c^-} f(x)$ ) or right side ( $\lim_{x \rightarrow c^+} f(x)$ ).
5. **Two-sided limit:** The limit describes the behavior from both sides (left and right-hand) of a point in a two-sided limit.
6. **Discontinuities:** In this type, the limit of a function does not exist at a specific point due to a discontinuity in the function. Some common types of discontinuities are given below:
  - **Jump Discontinuity:** A jump discontinuity occurs when the limits from a point’s left-hand and right-hand sides exist but both are not equal.
  - **Removable Discontinuity:** A removable discontinuity happens when the limit of a function at a certain point exists, but the function has a hole or gap in its graph at that point. A removable discontinuity occurs when the left-



hand and right-hand side limits of a function exist and both are equal at a specific point  $c$ , but the value of the function at that point ( $f(c)$ ), does not match the limit value.

- **Infinite Discontinuity:** It occurs when the limit of a function at a particular point does not exist. This can occur when the function reaches either positive or negative infinity.

Understanding and identifying these types of limits are important in analyzing the behavior of the function.

## Limit Rules

If  $\lim_{x \rightarrow c} f(x)$ , and  $\lim_{x \rightarrow c} g(x)$  exist then

**Addition and Subtraction rule:**  $\lim_{x \rightarrow c} (f(x) \pm g(x)) = \lim_{x \rightarrow c} (f(x)) \pm \lim_{x \rightarrow c} (g(x))$

**Constant multiple rule:**  $\lim_{x \rightarrow c} (k f(x)) = k \lim_{x \rightarrow c} (f(x))$  (where  $k$  is any constant)

**Product or multiple rule:**  $\lim_{x \rightarrow c} (f(x) * g(x)) = \lim_{x \rightarrow c} (f(x)) * \lim_{x \rightarrow c} (g(x))$

**Quotient rule**  $\lim_{x \rightarrow c} (f(x) / g(x)) = \lim_{x \rightarrow c} (f(x)) / \lim_{x \rightarrow c} (g(x))$  (where  $\lim_{x \rightarrow c} g(x) \neq 0$ )

**Exponent (or power) rule of limit**  $\lim_{x \rightarrow c} (f(x))^n = (\lim_{x \rightarrow c} f(x))^n$  (where  $n$  is any integer)

Try a [limit calculator with steps](#) to find the solution of the limit problems according to the above rules in few seconds.

## Method to determine an indeterminate form

Let's learn how to evaluate the indeterminate form such as  $0 / 0$  or  $\infty / \infty$ :

- **Algebraic technique:** start by simplifying the indeterminate form as much as possible. This can involve factoring, canceling common factors, or using algebraic manipulation to simplify the expression.
- **Rationalize:** If the indeterminate form involves a radical in the numerator or

denominator, we can rationalize the expression by multiplying both the numerator and denominator by a suitable conjugate to eliminate the radical.

- **L' Hopital's Rule:** Take the derivative of the numerator and the function's denominator to find the limit's indeterminate form. It is applicable when the limit of the form is  $0/0$  or  $\infty / \infty$ .
- **Limit of Trigonometric functions:** Trigonometric functions have specific limits. Like,  $\lim_{x \rightarrow 0} \sin x / x = 1$ , and  $\lim_{x \rightarrow 0} \cos x = 1$ .
- $\lim_{x \rightarrow 0} (1 - \cos x / x) = 1$
- $\lim_{x \rightarrow 0} (\sin x / x) = 1$
- $\lim_{x \rightarrow 0} (\tan x / x) = 1$
- **Exponential and logarithmic limits:** The exponent and logarithmic functions have specific limits.
- $\lim_{x \rightarrow 0} (1 + x)^{(1/x)} = e$
- $\lim_{x \rightarrow \infty} (e^x) = \infty$
- $\lim_{x \rightarrow -\infty} (e^x) = 0$

## Solution of some examples of Limit

### Example 1.

Calculate  $\lim_{x \rightarrow 1} [(x^3 + 4x^2 + x + 4) / (x + 4)]$

### Solution

$$\lim_{x \rightarrow 1} [(x^3 + 4x^2 + x + 4) / (x + 4)]$$

Substitute 1 into the given function,

$$= (1^3 + 4(1^2) + 1 + 4) / (1 + 4)$$

$$= 10 / 5 = 2$$

$$\text{Hence, } \lim_{x \rightarrow 1} [(x^3 + 4x^2 + x + 4) / (x + 4)] = 2$$

### Example 2.

Determine the  $\lim_{x \rightarrow 6} (x^2 - 6) / (x - 6)$

**Solution:**

$$\lim_{x \rightarrow 6} (x^2 - 36) / (x - 6) = (36 - 36) / 6 - 6 = 0 / 0$$

It is an indeterminate form. Therefore, we will solve it with algebraic techniques.

$$= \lim_{x \rightarrow 6} (x^2 - 6^2) / (x - 6)$$

$$\square (a - b)(a + b) = (a^2 - b^2)$$

$$= \lim_{x \rightarrow 6} (x - 6)(x + 6) / (x - 6)$$

After cancellation, we get

$$= \lim_{x \rightarrow 6} (x + 6)$$

Apply the limit

$$= 6 + 6 = 12$$

Hence, the  $\lim_{x \rightarrow 6} (x^2 - 36) / (x - 6) = 12$

## Conclusion

In this article, we have discussed the limit with its mathematical representation. We have discussed the different types of limits. Further, we examined the types of discontinuity limits. We explored the important rules of limit. We learned how to determine the limit of indeterminate form. We covered solutions of some examples of limits. After understanding this article, you can determine the limit of any function.